

EVALUATION OF HIGH-SPEED FILMS FOR MEASURING DECELERATIONS IN  
EXPERIMENTAL COLLISIONS

Paper presented to the Society of Photo-optical Instrumentation  
Engineer's Seminar in Depth and Equipment Demonstration:  
Optical instrumentation, A problem solving tool in automotive  
safety engineering and biomechanics, Dearborn, Mi., November  
20-21, 1972

R-72-5

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Voorburg, 1972

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## SUMMARY

High-speed filming of collisions of vehicles with e.g. obstacles can be used to yield quantitative information.

To get the deceleration history of the vehicle the series of positions read from the film should be differentiated twice.

As the positions are corrupted with noise, the differentiating should be done carefully in order to get a reliable signal.

The paper deals with how this can be arranged and gives the properties of the method.

One important parameter of the problem is the standard deviation of the error in the positions. A rule of thumb, employable for quantitative filming in general, is presented to estimate this parameter.

## Introduction

As part of an investigation into safety aspects of obstacles along the road experimental collisions between cars and obstacles have been carried out.

One of the goals of the project is to get an idea if passengers can survive such a collision and how to modify the obstacles to improve the situation.

Rather easily to measure is the deceleration of the car which can give useful information and at least will do to compare different modifications of one obstacle.

The deceleration can be measured with accelerometers mounted in the car.

Another possibility is high-speed filming of the car during the crash and processing the film to a deceleration curve. The paper deals with this processing.

Apparatus

The films have been made with an electrically driven stop motion Millikan camera which can run at 400 frames per second, loaded with a Kodak Four X positive 16 mm film. No artificial lighting has been used.

During the filming time marks are put on the edge of the film to yield the precise speed of filming afterwards. Special signs are painted on the side of the car to ease the reading out of the film. The film is read out with a motion analyser, type Boscar of Benson France.

For each frame a pair of x and y hairlines is put on a suitable marked point of the car and on a fixed reference point. The co-ordinates are punched directly into papertape, ready to be processed by a computer. Reading out time is about 3 seconds per point.

Processing positions to accelerations

The problem would be simple if positions did not have errors. However, the errors are quite large, e.g. a vehicle with a speed of 45 mph will move 2" in 1/300 s and the error in the position is about 20% of this distance.

Differentiating data with noise will strengthen the noise comparatively to the signal, so filtering is necessary. The problem is to find a suitable filter that smoothes out the noise to an acceptable level and keeps as much as possible of the signal.

Errors in position

The errors in the positions read out from the film were assumed to be random with zero mean, Gaussian distributed and independent for different filmframers.

These assumptions have been investigated and survived statistical testing. So the only parameter needed is the standard deviation of the errors. An estimation of this quantity can be made with the following rule of thumb.

$$\sigma = 0.41 S/n \quad (1)$$

with S= scale of filming

n= resolution power of the film expressed in a number of lines per unit of length.

In the factor 0.41 is included the effect that the position is the difference of two readings, one of the vehicle and one of the reference point.

Testing of the rule has proved that it somewhat underestimates the error with a factor between 1 and 2. In fact the rule expresses the fact that the resolution power of the film determines the accuracy of quantitative filming, under the condition that other things have been well chosen.

It can be applied to other situations, e.g. filming of the traffic flow on a road from the air with the goal to measure vehicle positions and velocities.

Differentiating process

Two aspects of the process will be considered:

- 1) the errors in the positions result into errors in the accelerations; the goal is to keep the standard deviation of the errors in the acceleration under a certain value.
- 2) the frequency response function of the process which describes what happens to harmonic components in the signal dependent on their frequency; the goal is to get a good frequency response from zero to a value as high as possible.

As positions are available as a time series a so-called difference scheme for the second derivative is needed. In general it has the form:

$$a_j = \sum_{k=-m}^m c_k x_{j+k} / \Delta t^2 \quad (2)$$

with  $\Delta t$  = time step

$a_j$  = acceleration at time  $j \Delta t$

$c_k$  = coefficient of the scheme

$x_{j+k}$  = position at time  $(j+k) \Delta t$

Coefficients  $c_k$  should be chosen so that the requirements are met. From equation (2) follows directly how the standard deviation of the acceleration,  $\sigma_a$ , depends on the st. dev. of the position,  $\sigma_x$

$$\sigma_a = \left[ \sum_{k=-m}^m c_k^2 \right]^{\frac{1}{2}} \sigma_x / \Delta t^2 \quad (3)$$

The frequency response function corresponding with equation (2) is (assuming a symmetric scheme i.e.  $c_{-k} = c_k$ )

$$H(w) = \left[ c_0 + 2 \sum_{k=1}^m c_k \cos(kw \Delta t) \right] / \Delta t^2 \quad (4)$$

So  $\sigma_a$  and  $H(w)$  both depend on the  $c_k$ 's and the parameters  $\sigma_x$  and  $\Delta t$ .

A direct relation between  $\sigma_a$  and  $H(w)$  exists in the following form: (application of Parseval's theorem):

$$\frac{\Delta t}{2\pi} \int_{-\pi/\Delta t}^{\pi/\Delta t} |H(w)|^2 dw = \sigma_a^2 / \sigma_x^2 \quad (5)$$

The ideal form of  $H(w)$  for differentiating twice is:

$$H(w) = -w^2 \quad (6)$$

Suppose one can realize this ideal form for a limited frequency band, i.e. (6) holds for  $|w| < w_0$  and  $H(w) = 0$  for  $|w| > w_0$ .

Substituting such an  $H(w)$  in equation (5) gives:

$$w_0^5 \Delta t = 5\pi \sigma_a^2 / \sigma_x^2 \quad (7)$$

From equation (7) can be seen that a decrease of the random error in the acceleration (decrease of  $\sigma_a$ ) will inevitably result in a frequency response function that is less ideal (decrease of  $w_0$ ) and vice versa. Also the influence of the time step  $\Delta t$  and the st. dev. of the error in positions can be evaluated. E.g. filming with two times a certain speed, i.e. halving  $\Delta t$ , keeping  $\sigma_a$  and  $\sigma_x$  fixed, will permit a 15% higher  $w_0$ ; and filming with a two times bigger film size, i.e. halving  $\sigma_x$ , keeping  $\Delta t$  and  $\sigma_a$  fixed, will permit a 32% higher  $w_0$ .



Application

Given were a time step  $\Delta t$  of  $1/400$  s, a scale of filming  $S$  of about 1750, a resolution power of the film of 2550 lines/inch and an acceptable st. dev. of random error in acceleration of 1g.

The st. dev. of the error in the positions is estimated with equation (1) at .28". Empirical determination yielded 0.4", i.e. 40% more.

Using equation (7) yields for the maximum frequency  $w_0$  a value of 90 rad/s = 14.3 c/s.

In fact the ideal response function of equation (6), even for a limited frequency range, can only be approximated. Here it has been done with a difference scheme of 3 steps:

$$y_j = [19x_j + 16(x_{j-1} + x_{j+1}) + 10(x_{j-2} + x_{j+2}) + 4(x_{j-3} + x_{j+3}) + (x_{j-4} + x_{j+4})] / 81 \quad (8)$$

$$v_j = (-3y_{j-6} - 2y_{j-4} - y_{j-2} + y_{j+2} + 2y_{j+4} + 3y_{j+6}) / (56\Delta t) \quad (9)$$

$$a_j = (-3v_{j-6} - 2v_{j-4} - v_{j-2} + v_{j+2} + 2v_{j+4} + 3v_{j+6}) / (56\Delta t) \quad (10)$$

Equation (8) only smoothes the data and filters out frequencies above 100c/s.

Equation (8) and (9) yield velocities and accelerations, they are derived from the least squares fit of a polynomial of second degree to 7 points.

The three equations can be combined into one of the type of equation (2)

The properties of the processing are:  $\sigma_a = 1.3$  g, i.e. 30% too high and

$$T(w) = H(w) / (-w^2) =$$

$$\left[ \frac{(1+2\cos w\Delta t)}{3} \right]^4 \left[ 9\sin 6w\Delta t + 6\sin 4w\Delta t + 3\sin 2w\Delta t \right]^2 / (84\Delta t)^2 \quad (11)$$

Instead of  $H(w)$  itself  $T(w)$  has been chosen to represent the response aspects of the process.  $T(w)$  should approximate 1 for  $w < w_0$  and 0 for  $w > w_0$ ; see Figure 1.

The approximation is not so good and work is in progress to improve it.

However, practical results seem to be fairly good. Films of collisions with lighting poles, poles for emergency phones and crash cushions have been processed; see Figure 2 for an example.

Final remarks

High-speed filming as a tool to get the deceleration curve of a colliding vehicle has several properties.

Among the negative ones are:

- the frequencies in the signal that can be handled are rather limited, 10 to 20 c/s;
- the method depends on favourable lighting conditions.

Among the positive ones are:

- high reliability because the measuring system cannot be affected by the collision;
- the film can also be used for qualitative inspection of the collision;
- when a modern motion analyser with computer compatible output is available the method is rather cheap.

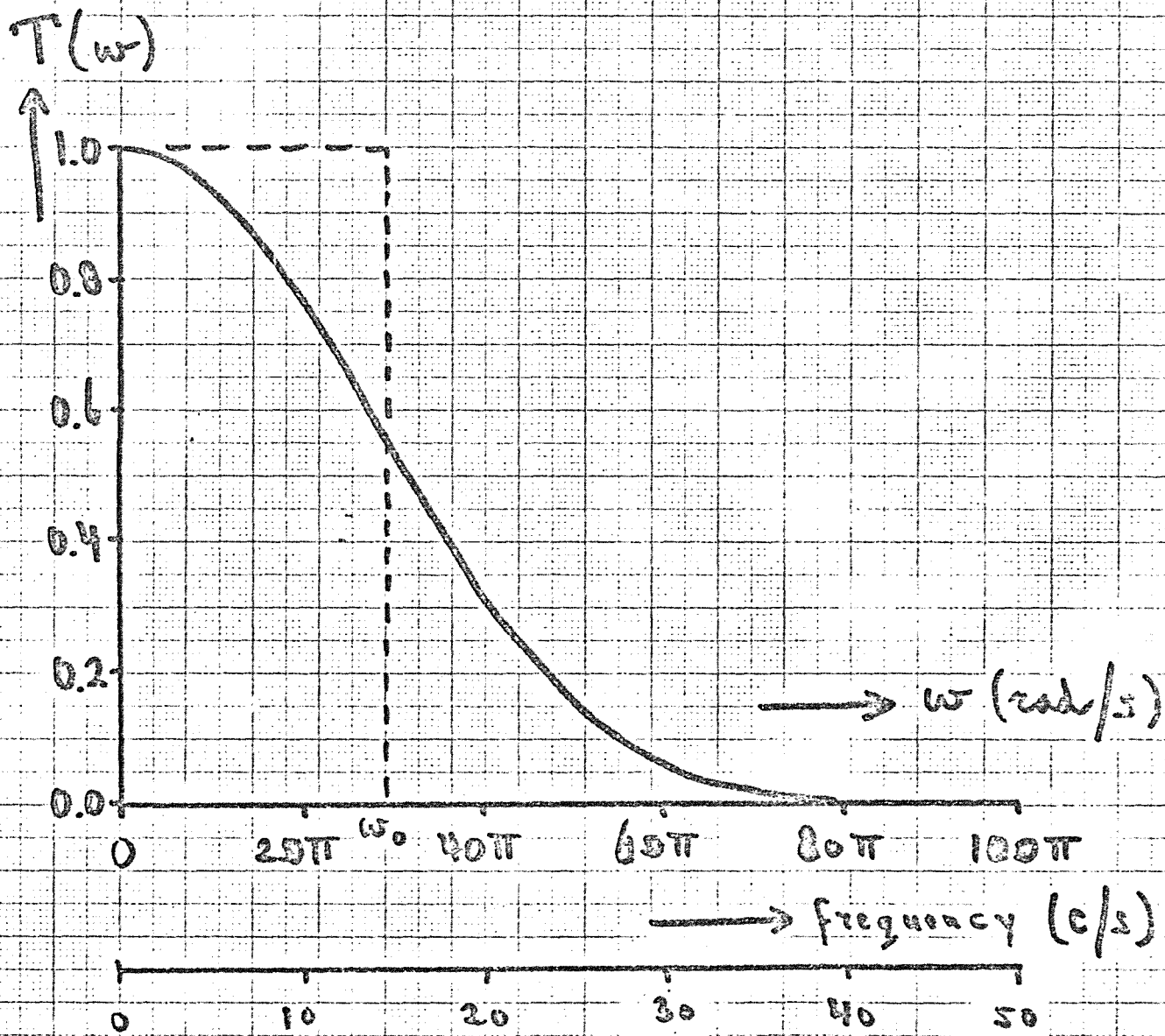


Figure 1

# Lighting pole

Proef nummer : L 5.  
Masttype : Fe 10m.  
Botsmethode : 105 km/h.  
Snelheid t=185 msec : ~~44 km/h~~

Velocity before collision: 65 mph  
Velocity at time = 185 ms: 25 mph

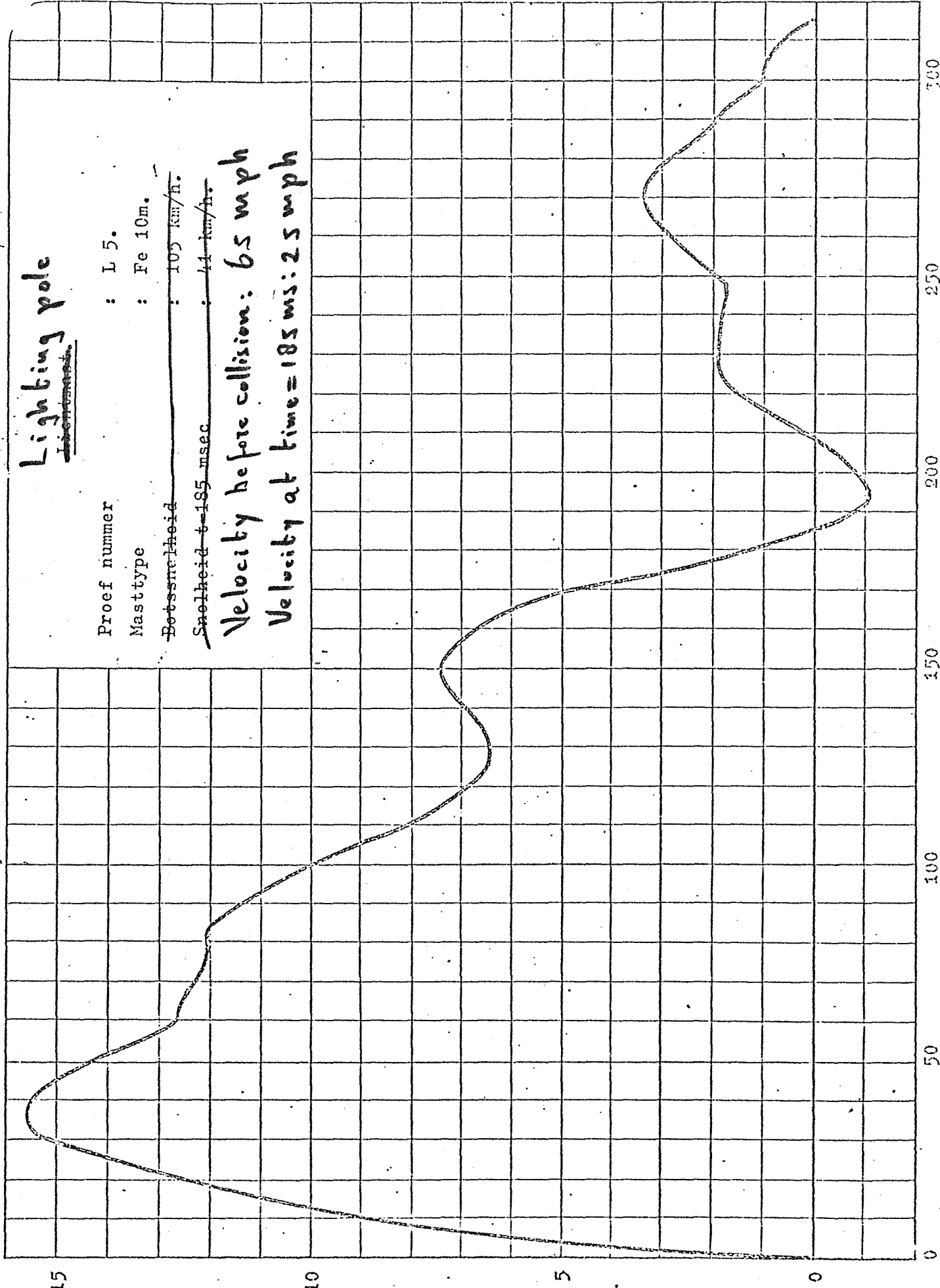


Figure 2